$\boxed{3B}$ Continue

To summarize from Equations (2-1), (2-5), (3-3), (3-4) and (3-5) as Equations (3-6) through (3-15):

$$\frac{\partial Gain}{\partial TFG} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ \frac{1.0}{TFG} \tag{3-6}$$

$$\frac{\partial Phase}{\partial TFG} = 0.0 \tag{3-7}$$

$$\frac{\partial Gain}{\partial b_1{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ Re\left[\frac{s}{N^1}\right] \tag{3-8}$$

$$\frac{\partial Phase}{\partial b_1{}^1} = \frac{W\ (180.0/\pi\ )}{N_p}\ Im\left[\frac{s}{N^1}\right] \tag{3-9}$$

$$\frac{\partial Gain}{\partial b_0{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ Re\left[\frac{1.0}{N^1}\right] \tag{3-10}$$

$$\frac{\partial Phase}{\partial b_0{}^1} = \frac{W\ (180.0/\pi\ )}{N_p}\ Im\left[\frac{1.0}{N^1}\right] \tag{3-11}$$

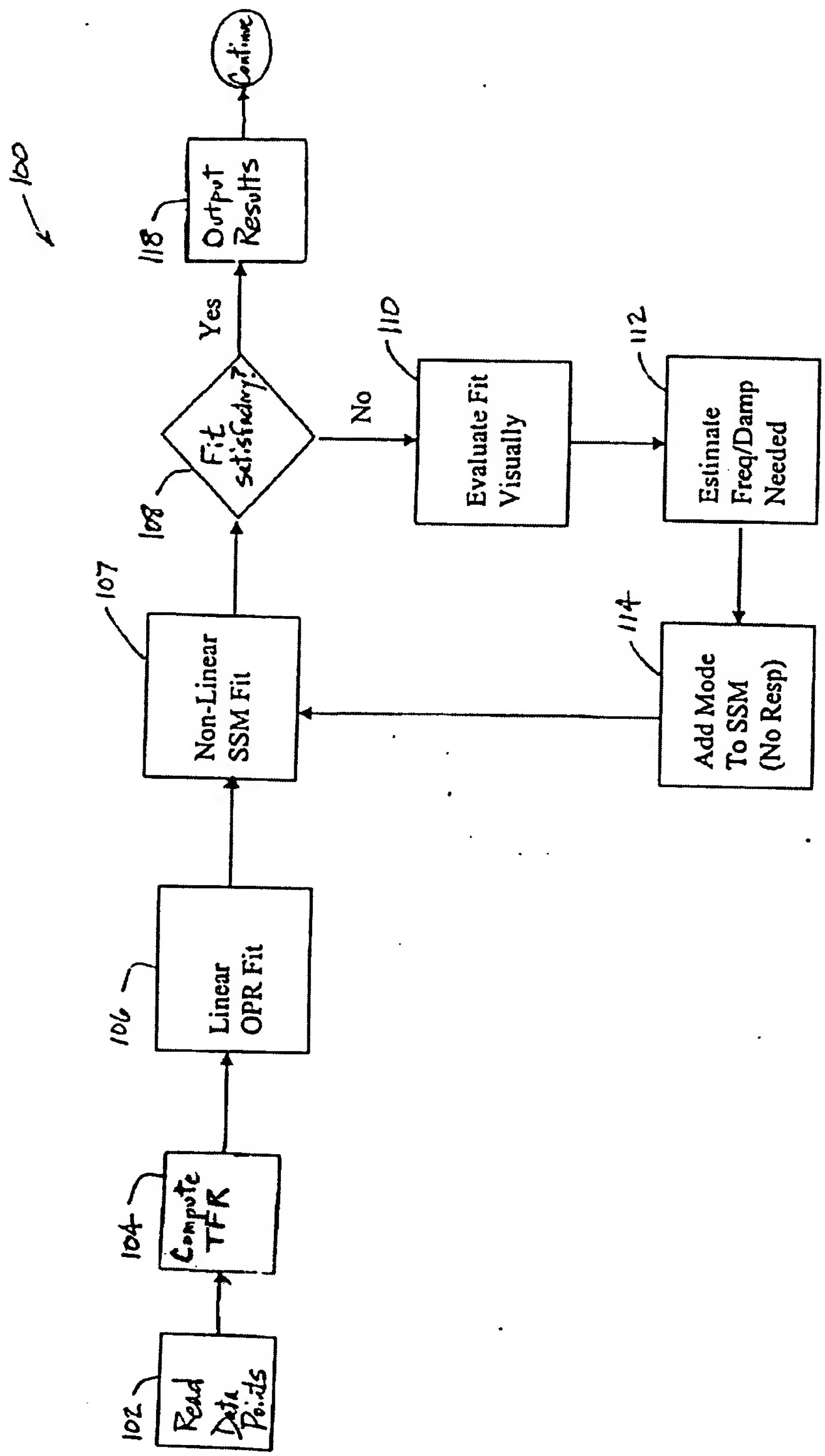
$$\frac{\partial Gain}{\partial a_1{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ Re\left[\frac{-s}{D^1}\right] \tag{3-12}$$

$$\frac{\partial Phase}{\partial a_1{}^1} = \frac{W\ (180.0/\pi\ )}{N_p}\ Im\left[\frac{-s}{D^1}\right] \tag{3-13}$$

$$\frac{\partial Gain}{\partial a_0{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ Re\left[\frac{-1.0}{D^1}\right] \tag{3-14}$$

$$\frac{\partial Phase}{\partial a_0{}^1} = \frac{W\ (180.0/\pi\ )}{N_p}\ Im\left[\frac{-1.0}{D^1}\right] \tag{3-15}$$

Fig. 3B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102
Read Data Points
104
Compute TFR
106
Linear OPR Fit
107
Non-Linear SSM Fit
108
Fit satisfactory?
Yes
118
Output Results
Continue
No
110
Evaluate Fit Visually
112
Estimate Freq/Damp Needed
114
Add Mode To SSM (No Resp)

Fig. 1

$$\frac{\partial Gain}{\partial x} = \frac{\partial\ (\ W/N_g\ 20.0\ \log_{10}(|Z|)\ )}{\partial x} \tag{2-1}$$

$$\frac{\partial Phase}{\partial x} = \frac{\partial\ (\ W/N_p\ (180.0/\pi)\ \tan^{-1}(Im(Z)/Re(Z))\ )}{\partial x} \tag{2-2}$$

Where: Gain = gain of transfer function response in dB
Phase = phase of transfer function response in degrees
W = frequency dependent weighting
Ng = gain normalization
Np = phase normalization
Z = complex transfer function frequency response
x = design variable

Since: $|Z| = \sqrt{Z\ Z^*}$

$\log_{10}(u) = \log_{10}(e)\ln(u)$

Gives: $20.0\log_{10}(|Z| = 10.0\log_{10}(e)\ln(Z\ Z^*)$

Then: $$\frac{\partial Gain}{\partial x} = \frac{\partial\ (\ W/Ng\ 10.0\log_{10}(e)\ \ \ln(Z\ Z^*)}{\partial x}$$

Since: $$\frac{\partial \ln(u)}{\partial x} = \frac{1.0}{u}\frac{\partial u}{\partial x}$$

$$\frac{\partial \tan^{-1}(u)}{\partial x} = \frac{1.0}{1.0+u^2}\frac{\partial u}{\partial x}$$

Then: $$\frac{\partial Gain}{\partial x} = \frac{W\ 10.0\log_{10}(e)}{N_g\ \ (Re(Z)^2+Im(Z)^2)}\ \frac{\partial(Re(Z)^2+Im(Z)^2)}{\partial x}$$

$$\frac{\partial Phase}{\partial x} = \frac{W\ (180.0/\pi)\ Re(Z)^2}{N_p\ \ (Re(Z)^2+Im(Z)^2)}\ \frac{\partial(Im(Z)/Re(Z))}{\partial x}$$

Since: $$\frac{\partial(u/v)}{\partial x} = \frac{1.0}{v^2}\left(v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x}\right)$$

Gives: $$\frac{\partial Gain}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_p\ \ (Re(Z)^2+Im(Z)^2)}\left(Re(Z)\frac{\partial Re(Z)}{\partial x} + Im(Z)\frac{\partial Im(Z)}{\partial x}\right)$$

$$\frac{\partial Phase}{\partial x)} = \frac{W\ (180.0/\pi)}{N_p\ \ (Re(Z)^2+Im(Z)^2)}\left(Re(Z)\frac{\partial Im(Z)}{\partial x} - Im(Z)\frac{\partial Re(Z)}{\partial x}\right)$$

Continue to 2B

Fig. 2A

2B Continue

There is similarity between the partial of the gain of the response and that of the phase. To uncover the similarity, examine Equation (2-3):

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{Re(Z)+Im(Z)j}\left[\frac{\partial Re(Z)}{\partial x} + \frac{\partial Im(Z)}{\partial x}j\right] \tag{2-3}$$

Gives:

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{(Re(Z)^2+Im(Z)^2)}\left[Re(Z)\frac{\partial Re(Z)}{\partial x} + Im(Z)\frac{\partial Im(Z)}{\partial x}\right] + \frac{1.0}{(Re(Z)^2+Im(Z)^2)}\left[Re(Z)\frac{\partial Im(Z)}{\partial x} - Im(Z)\frac{\partial Re(Z)}{\partial x}\right]j$$

Combining the results from Equations (2-1), (2-2) and (2-3) yield Equations (2-4) and (2-5):

$$\frac{\partial Gain}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_g} Re\left[\frac{1.0}{z}\frac{\partial z}{\partial x}\right] \tag{2-4}$$

$$\frac{\partial Phase}{\partial x} = \frac{W\ (180.0/\pi)}{N_p} Im\left[\frac{1.0}{z}\frac{\partial z}{\partial x}\right] \tag{2-5}$$

The complex response of the block diagonal SSM for a specific transfer function is given by Equation (2-6):

$$z_{ij} = \Sigma\left[\frac{N_{ij}{}^{1}}{D^{1}}\right] + d_{ij} \tag{2-6}$$

Where:

$$N_{ij}{}^{1} = (c_{i1}{}^{1}\ b_{1j}{}^{1} + c_{i2}{}^{1}\ b_{2j}{}^{1})\ s + (c_{i2}{}^{1}\ b_{1j}{}^{1}\ a_{21}{}^{1} - c_{i1}{}^{1}\ b_{1j}{}^{1}\ a_{22}{}^{1} + c_{i1}{}^{1}\ b_{2j}{}^{1})$$

$$D^{1} = s^{2} - a_{22}{}^{1}\ s - a_{21}{}^{1}$$

For elements in the D matrix the unknown term in Equations (2-4) and (2-5) is given by Equation (2-7) using Equation (2-6):

$$\frac{\partial z_{ij}}{\partial d_{ij}} = 1.0 \tag{2-7}$$

For elements in the A, B or C matrices, x1, the unknown term in Equations (2-4) and (2-5) is given by Equation (2-8):

$$\frac{\partial z_{ij}}{\partial x^{1}} = \frac{\partial\ (N_{ij}{}^{1}/D_{1})}{\partial x^{1}} \tag{2-8}$$

Continue to 2C

Fig. 2B

2C Continue

Since: $$\frac{\partial(u/v)}{\partial x} = \frac{1.0}{v^2}\left[v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x} \right]$$

Then: $$\frac{\partial z_{ij}}{\partial x^1} = \frac{1.0}{D^1 D^1}\left[D^1\frac{\partial N_{ij}{}^1}{\partial x^1} - N_{ij}{}^1\frac{\partial D^1}{\partial x^1} \right]$$

And thus: $$\frac{\partial D^1}{\partial c_{i1}{}^1} = \frac{\partial D^1}{\partial c_{i2}{}^1} = \frac{\partial D^1}{\partial b_{1j}{}^1} = \frac{\partial D^1}{\partial b_{2j}{}^1} = 0.0$$

Simplified: $$\frac{\partial z_{ij}}{\partial x_1} = \frac{1.0}{D^1}\left(\frac{\partial N_{ij}{}^1}{\partial x^1} \right) \quad \text{for } x^1 = c_{i1}{}^1, c_{i2}{}^1, b_{1j}{}^1, b_{2j}{}^1$$

From Equation (2-6) the non-zero partials of the block numerator and denominator are given as Equations (2-9) through (2-16):

$$\frac{\partial N_{ij}{}^1}{\partial c_{i1}{}^1} = b_{1j}{}^1\, s + (b_{2j}{}^1 - b_{1j}{}^1\, a_{22}{}^1) \tag{2-9}$$

$$\frac{\partial N_{ij}{}^1}{\partial c_{i2}{}^1} = b_{2j}{}^1\, s + (b_{1j}{}^1\, a_{21}{}^1) \tag{2-10}$$

$$\frac{\partial N_{ij}{}^1}{\partial b_{1j}{}^1} = c_{i1}{}^1\, s + (c_{i2}{}^1\, a_{21}{}^1 - c_{i1}{}^1\, a_{22}{}^1) \tag{2-11}$$

$$\frac{\partial N_{ij}{}^1}{\partial b_{2j}{}^1} = c_{i2}{}^1\, s + (c_{i1}{}^1) \tag{2-12}$$

$$\frac{\partial N_{ij}{}^1}{\partial a_{21}{}^1} = c_{i2}{}^1\, b_{1j}{}^1 \tag{2-13}$$

$$\frac{\partial N_{ij}{}^1}{\partial a_{22}{}^1} = -\, c_{i1}{}^1\, b_{1j}{}^1 \tag{2-14}$$

$$\frac{\partial D^1}{\partial a_{21}{}^1} = -\, 1.0 \tag{2-15}$$

$$\frac{\partial D^1}{\partial a_{22}{}^1} = -\, s \tag{2-16}$$

Continue to 2D

Fig. 2C

2D Continue

To summarize from Equations (2-4) through (2-16) as Equations (2-17) through (2-30):

$$\frac{\partial Gain_{ij}}{\partial a_{21}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{D^{1}c_{i2}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}}{D^{1}D^{1}Z_{ij}}\right] \quad (2\text{-}17)$$

$$\frac{\partial Phase_{ij}}{\partial a_{21}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{D^{1}c_{i2}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}}{D_{1}D_{1}Z_{ij}}\right] \quad (2\text{-}18)$$

$$\frac{\partial Gain_{ij}}{\partial a_{22}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{-D^{1}c_{i1}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}s}{D^{1}D^{1}Z_{ij}}\right] \quad (2\text{-}19)$$

$$\frac{\partial Phase_{ij}}{\partial a_{22}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{-D^{1}c_{i1}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}s}{D^{1}D^{1}Z_{ij}}\right] \quad (2\text{-}20)$$

$$\frac{\partial Gain_{ij}}{\partial b_{1j}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{c_{i1}{}^{1}s+c_{i2}{}^{1}a_{21}{}^{1}-c_{i1}{}^{1}a_{22}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}21)$$

$$\frac{\partial Phase_{ij}}{\partial b_{1j}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{c_{i1}{}^{1}s+c_{i2}{}^{1}a_{21}{}^{1}-c_{i1}{}^{1}a_{22}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}22)$$

$$\frac{\partial Gain_{ij}}{\partial b_{2j}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{c_{i2}{}^{1}s+c_{i1}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}23)$$

$$\frac{\partial Phase_{ij}}{\partial b_{2j}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{c_{i2}{}^{1}s+c_{i1}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}24)$$

$$\frac{\partial Gain_{ij}}{\partial c_{i1}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{b_{1j}{}^{1}s+b_{2j}{}^{1}-b_{1j}{}^{1}a_{22}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}25)$$

$$\frac{\partial Phase_{ij}}{\partial c_{i1}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{b_{1j}{}^{1}s+b_{2j}{}^{1}-b_{1j}{}^{1}a_{22}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}26)$$

$$\frac{\partial Gain_{ij}}{\partial c_{i2}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{b_{2j}{}^{1}s+b_{1j}{}^{1}a_{21}{}^{1}}{D^{1}Z_{ij}}\right] \quad (2\text{-}27)$$

$$\frac{\partial Phase_{ij}}{\partial c_{i2}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{b_{2j}{}^{1}s+b_{1j}{}^{1}a_{21}{}^{1}}{D^{1}\ Z_{ij}}\right] \quad (2\text{-}28)$$

$$\frac{\partial Gain_{ij}}{\partial d_{ij}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \operatorname{Re}\left[\frac{1.0}{Z_{ij}}\right] \quad (2\text{-}29)$$

$$\frac{\partial Phase_{ij}}{\partial d_{ij}} = \frac{W\ (180.0/\pi)}{N_{pij}} \operatorname{Im}\left[\frac{1.0}{Z_{ij}}\right] \quad (2\text{-}30)$$

Fig. 2D

The complex response of the PZM is given by Equation (3-1):

$$Z = \frac{N}{D} = \frac{TFG \prod N^1}{\prod D^1} \quad (3\text{-}1)$$

Where: $N^1 = s^2 + b_1{}^1 s + b_0{}^1$

$D^1 = s^2 + a_1{}^1 s + a_0{}^1$

The unknown term in Equations (2-4) and (2-5) is given by Equation (3-2) by using Equation (3-1):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{D\,N}\left[D\frac{\partial N}{\partial x} - N\frac{\partial D}{\partial x} \right] \quad (3\text{-}2)$$

The results of Equation (3-2) when the transfer function gain is the design variable, x, is given by Equation (3-3).

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{TFG} \quad \text{when } x = TFG \quad (3\text{-}3)$$

The results of Equation (3-2) when the a numerator block coefficient is the design variable, x, is given by Equation (3-4):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N}\left(\frac{\partial N}{\partial x} \right) = \frac{1.0}{N^1}\frac{\partial N^1}{\partial x} \quad \text{when } x = b_1{}^1 \text{ or } b_0{}^1 \quad (3\text{-}4)$$

Gives: $\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1} s \quad \text{when } x = b_1{}^1$

$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1} \quad \text{when } x = b_0{}^1$

The results of Equation (3-2) when the a denominator block coefficient is the design variable, x, is given by Equation (3-5):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D}\left(\frac{\partial D}{\partial x} \right) = -\frac{1.0}{D^1}\frac{\partial D^1}{\partial x} \quad \text{when } x = a_1{}^1 \text{ or } a_0{}^1 \quad (3\text{-}5)$$

Gives: $\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1} s \quad \text{when } x = a_1{}^1$

$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1} \quad \text{when } x = a_0{}^1$

Continue to $\boxed{3B}$

Fig. 3A

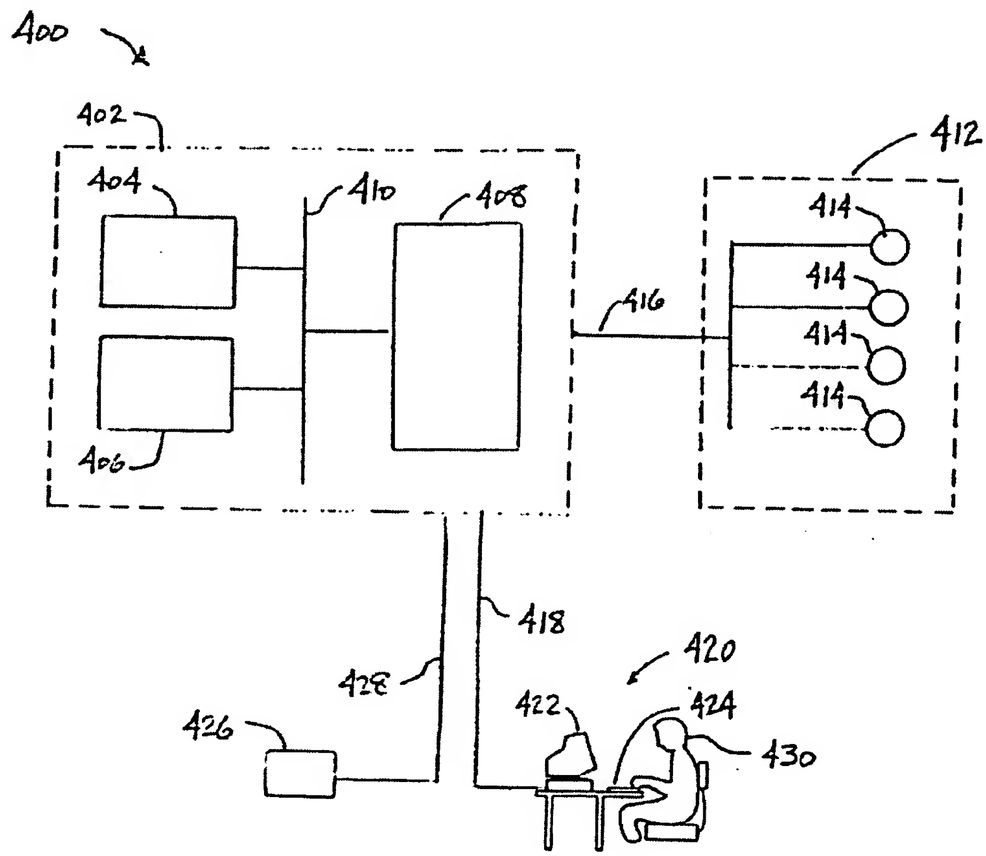
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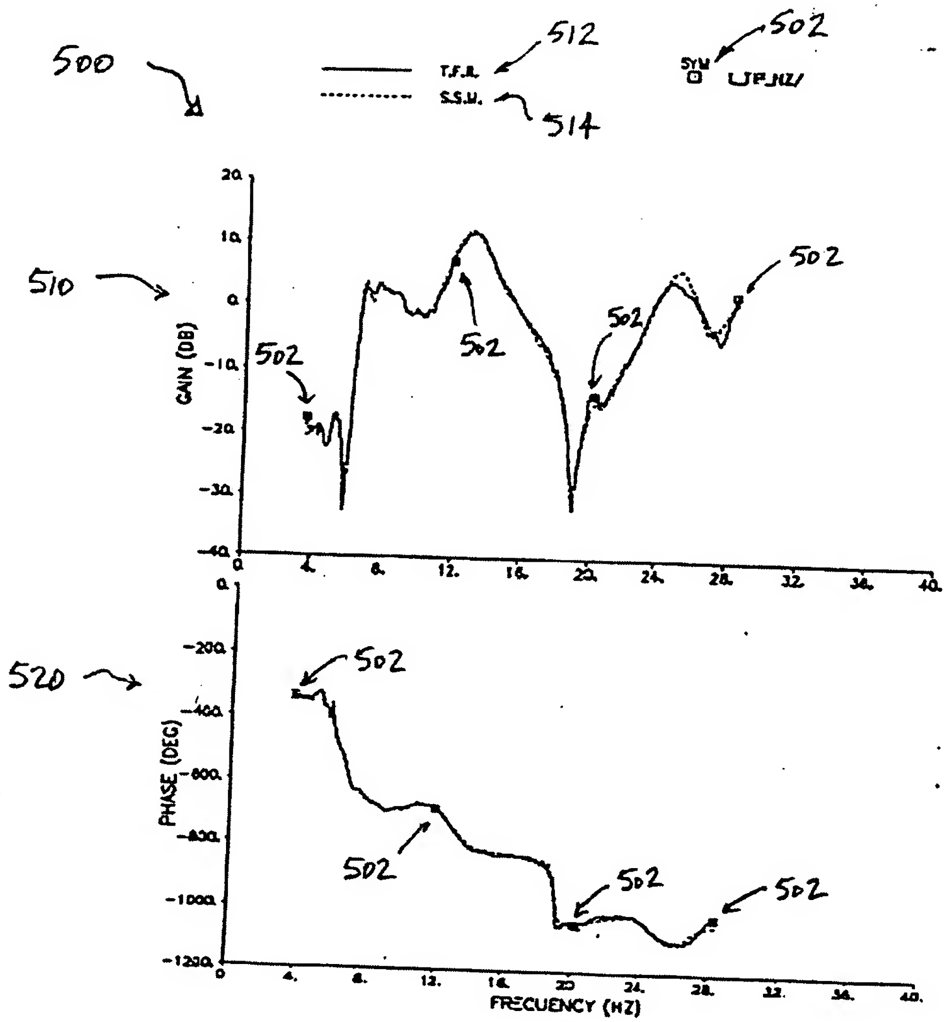
Fig. 4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512
514
502
T.F.R.
S.S.U.
SYM
510
520
GAIN (DB)
PHASE (DEG)
FRECUENCY (HZ)

Fig. 5